Теория:

1) $\sin\alpha\cdot\sin\beta=\frac{1}{2}(\cos(\alpha-\beta)-\cos(\alpha+\beta))$;

2) $\cos\alpha\cdot\cos\beta=\frac{1}{2}(\cos(\alpha-\beta)+\cos(\alpha+\beta))$;

3) $\sin\alpha\cdot\cos\beta=\frac{1}{2}(\sin(\alpha+\beta)+\sin(\alpha-\beta))$.

Пример. $\mathrm{ctg}\,10^\circ\,\mathrm{ctg}\,50^\circ\,\mathrm{ctg}\,70^\circ=\frac{\cos10^\circ\cos50^\circ\cos70^\circ}{\sin10^\circ\sin50^\circ\sin70^\circ}=$
$=\frac{\frac{1}{2}(\cos60^\circ+\cos40^\circ)\cos70^\circ}{\frac{1}{2}(\cos40^\circ-\cos60^\circ)\sin70^\circ}=\frac{\frac{1}{4}\cos70^\circ+\frac{1}{4}(\cos30^\circ+\cos110^\circ)}{\frac{1}{4}(\sin110^\circ+\sin30^\circ)-\frac{1}{4}\sin70^\circ}=\frac{\cos70^\circ-\cos70^\circ+\frac{\sqrt{3}}{2}}{\sin70^\circ+\frac{1}{2}-\sin70^\circ}=$
$=\sqrt{3}$.

Пример. Упростить $\sin\frac{\pi}{5}+\sin\frac{2\pi}{5}+\sin\frac{3\pi}{5}+\sin\frac{4\pi}{5}$.
Путь $\sin\frac{\pi}{5}+\sin\frac{2\pi}{5}+\sin\frac{3\pi}{5}+\sin\frac{4\pi}{5}=A$.
Домножим обе части равенства на $2\sin\frac{\pi}{5}$.
$2\sin^2\frac{\pi}{5}+2\sin\frac{\pi}{5}\sin\frac{2\pi}{5}+2\sin\frac{\pi}{5}\sin\frac{3\pi}{5}+2\sin\frac{\pi}{5}\sin\frac{4\pi}{5}=2\sin\frac{\pi}{5}\cdot A$.
$1-\cos\frac{2\pi}{5}+\cos\frac{\pi}{5}-\cos\frac{3\pi}{5}+\cos\frac{2\pi}{5}-\cos\frac{4\pi}{5}+\cos\frac{3\pi}{5}-\cos\frac{5\pi}{5}=2\sin\frac{\pi}{5}\cdot A$.
$2+2\cos\frac{\pi}{5}=2\sin\frac{\pi}{5}\cdot A\quad\Leftrightarrow A=\frac{1+\cos\frac{\pi}{5}}{\sin\frac{\pi}{5}}=\frac{2\cos^2\frac{\pi}{10}}{2\sin\frac{\pi}{10}\cos\frac{\pi}{10}}=\mathrm{ctg}\,\frac{\pi}{10}$.